OPEN ACCESS JOURNALS
**Indiana Journal of Economics and Business Management**
*Abbriviate Tittle- Ind J Econ Bus Manag*
ISSN *(Online)*- 2583-3758
Journal Homepage Link- https://indianapublications.com/journal/IJEBM
DOI: https://doi.org/10.5281/zenodo.10397584

INDIANA PUBLICATIONS
PRODUCTIVE AND QUALITY RESEARCH

## Review Article

Volume-03|Issue-06-2023

# A Mathematical Model Integrating Environmental Economics and the UN Sustainable Development Goals

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**Article History**
Received: 28.11.2023
Accepted: 02.12.2023
Published: 16.12.2023

**Citation**
Muhammad, A., Jeroh, E., Ebitomi, T., Paulina Shittu Gaude- Jiwul, P. S., Abasido, A. U. (2023). A Mathematical Model Integrating Environmental Economics and the UN Sustainable Development Goals. *Indiana Journal of Economics and Business Management, 3*(6),43-48.

**Abstract:** Achieving sustainable development requires holistically balancing economic growth, social welfare, and environmental sustainability. This paper presents a conceptual mathematical model integrating environmental economics theory and the UN Sustainable Development Goals to determine optimal sectoral investments for maximizing sustainability. The model combines nested CES production functions depicting complex economic interdependencies with constraints on emissions, resource depletion, and investments in human/infrastructure capital aligned with SDG targets. An optimization framework then reveals trade-offs across competing economic, social, and environmental priorities to guide sustainable policymaking. While theoretical, the integrated structure provides a foundation to formally assess the complex dynamics between development pathways, ecological limits, and human well-being. Extensions incorporating innovation processes, equity considerations, climate impacts, validation, and uncertainty could enable practical application to inform integrated investment planning and sustainability transitions. Overall, this model offers a launching point to quantitatively analyze the intricate connections between multifaceted aspects of sustainable development.

**Keywords:** sustainable development, mathematical modeling, environmental economics, optimization, social welfare, Sustainable Development Goals.

## INTRODUCTION

### Background on sustainable development and the UN SDGs

Sustainable development has become a critical challenge in the 21st century in the face of growing resource constraints and environmental degradation. The United Nations Sustainable Development Goals (SDGs) outline 17 goals and 169 targets for achieving economic development, social inclusion, and environmental sustainability by 2030 (United Nations, 2015). However, trade-offs frequently exist between competing economic, social, and environmental priorities, requiring careful analysis to enable optimal resource allocation. Mathematical modeling can provide valuable insights for sustainably managing complex systems and guiding policy decisions. This article presents a mathematical model integrating environmental economics and the UN SDGs to determine optimal resource allocations for maximizing sustainable development.

Environmental economics applies microeconomic principles to balance economic activities with environmental considerations (Perman *et al.*, 2011). Various mathematical models have been formulated to characterize the complex dynamics between economic growth, resource utilization, and pollution emissions (Xepapadeas, 2010). Integrating these relationships with metrics for social welfare and environmental sustainability aligned with the SDGs can help determine how to best allocate limited resources. However, existing models remain limited in simultaneously capturing economic, social, and environmental dimensions.

This paper aims to address these gaps by combining economic production functions with environmental constraints and indicators for SDG achievement. The model optimization will determine sector-specific investments and interventions to maximize sustainable development given finite resources. By integrating across disciplinary perspectives, the model can provide cohesive, quantitative insights to help achieve the SDGs. This paper derivates the equations for the mathematical model and discusses the significance of the relationships. Further validation and refinement could allow application to specific countries or contexts.

## LITERATURE REVIEW

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### Existing research on mathematical models for sustainable development

A range of mathematical models have been developed within environmental economics to analyze the complex dynamics between economic growth, resource use, and environmental externalities. Seminal models include the Solow-Swan model exploring how technological change can offset diminishing returns from finite resources (Solow, 1956; Swan, 1956). Ramsey-type optimal growth models examine consumption, savings, and investment decisions to maximize intertemporal social welfare (Ramsey, 1928). Meanwhile, integrated assessment models (IAMs) incorporate interactions between socioeconomic and biophysical systems, such as linking economic activity to greenhouse gas emissions and climate change impacts (Nordhaus, 1992; Stern, 2007).

However, existing models tend to focus on individual aspects of sustainability in isolation rather than balancing economic, social, and environmental considerations simultaneously. For example, production functions in many growth models capture physical and human capital without accounting for impacts on natural capital (Löfgren & Thanh, 2005). Environmental constraints like pollution emissions may be included but not aligned with indicators for social welfare or sustainability goals (Annabi *et al.*, 2006). Attempts to incorporate equity weighting in welfare calculations also remain limited (Anthoff & Tol, 2010).

Furthermore, while IAMs model complex system dynamics, they remain highly aggregate and simplified representations spanning long time horizons, limiting their direct policy applicability (Pindyck, 2017). This inhibits quantitative analysis of sectoral investments or policy interventions to support near-term sustainable development. Linking model variables to measurable targets like the SDGs has also been underexplored. Therefore, key gaps persist in mathematical models that can holistically capture economic, social, and environmental dimensions of sustainability at relevant spatial and temporal scales.

An integrated model incorporating production functions, environmental economics, and SDG indicators could help address these limitations. Mapping model variables to the 17 SDGs and 169 underlying targets can ground the model in policy-relevant sustainability metrics to guide decision-making (Allen *et al.*, 2018). The model optimization can then determine cross-sectoral investments and interventions to maximize sustainable development subject to economic, social, and environmental constraints. Capturing these complex dynamics can provide data-driven and scientifically grounded evidence to support the 2030 Agenda for Sustainable Development.

## MODEL DERIVATION

### Complex Derivation of a Multifaceted Economic-Environmental Model

The theoretical foundation of an intricate economic-environmental model is established through a sophisticated framework amalgamating diverse parameters and relationships. Within this construct lie multifarious elements such as capital stock (K) segregated by numerous industry sectors (Ki), labor force (L) differentiated by varying skill levels (Lj), the Human Capital Index (H) founded on educational and health metrics, a vector of natural resource stocks (R), a vector representing pollutant emissions (E), among others.

### Defining Key Parameters:

- The model's complexity is underpinned by a constellation of parameters:
- Y represents Gross Domestic Product (GDP),
- K signifies the array of Capital Stock categorized across i industry sectors (Ki),
- L embodies the Labor force segmented by j skill levels (Lj),
- H encapsulates the Human Capital Index reliant on education and health,
- R signifies the Vector of m natural resource stocks (Rm),
- E represents the Vector of n pollutant emissions (En ),
- U stands for the Utility function responsible for social welfare optimization,
- I represents the Matrix of investments into capital, labor, and human capital,
- A symbolizes Total Factor Productivity,
- α,β,γ,δ embody Output Elasticities,
- η,θ,ρ,σ denote Environmental and Depreciation Parameters,
- t signifies the Time Period. (Stern, 2004; United Nations, 2021)

### Mathematical Formulation:

The cornerstone of this complex model is the nested Constant Elasticity of Substitution (CES) production function, delineating GDP Y as a function of capital, labor, human capital, and natural resources:

$$Y = A \cdot f(K1, \ldots, Ki, \ldots, KN, L1, \ldots, Lj, \ldots, LM, H, R1, \ldots, Rm)$$

Here, the function $f(K, L, H, R)$ incorporates intricate relationships by adopting a multi-dimensional structure:

$$f(K, L, H, R) = [\alpha \sum K\rho + \beta \sum L\rho + \gamma H\rho + \delta \sum R\rho]1/\rho$$

The function elucidates the convoluted interplay of capital, labor, human capital, and resources, each bearing specific elasticities (α,β,γ,δ) contributing to the intricacies of economic production.

### Damage Function and Social Welfare Maximization:

Amidst the complexity, the model integrates a damage function to account for the cumulative impact of multiple pollutants:

$$2D = \sum \theta n \cdot En2$$

Moreover, the crux of societal well-being (U(t)) in temporal t is derived from the difference between GDP (Y(t)) and environmental degradation (D(t)):

$$U(t) = Y(t) - D(t)$$

### Complex Constraints and Optimization:

A myriad of constraints, encompassing capital, labor, human capital, natural resource accumulation, emissions generation, and investment limitations, are characterized by intricate differential equations:

$$\begin{aligned} K^{\cdot} i = Ii - \delta Ki\, L^{\cdot} j = nLj - mLj\, H^{\cdot} = IH - \lambda H\, R^{\cdot} m \\ = -\rho Rm\, E^{\cdot} n = \sigma nY - \phi nEn \end{aligned}$$

Furthermore, a constraint tot ΣIi ≤ Itot regulates the total investment across sectors.

### Optimal Solution and Complexity Analysis:

The optimization over a time horizon, aiming to allocate totItot across various sectors, necessitates sophisticated mathematical optimization techniques. Dynamic programming or numerical methodologies are deployed to discern the optimal allocation while navigating the intricacies of constraints and multi-dimensional dependencies.

This elaborate model embodies the intricate nexus between economic production, environmental impact, resource allocation, and societal welfare. Its complexity necessitates sophisticated mathematical formulations and advanced optimization strategies for comprehensive analysis, policy formulation, and strategic decision-making.

### Stating Simplifying Assumptions

### Simplifying Assumptions for the Multifaceted Economic-Environmental Model

In refining the intricate economic-environmental model to render it more manageable and analytically approachable, several simplifying assumptions are introduced. These assumptions serve to streamline the complexities inherent in the model, providing a clearer framework for analysis, albeit at the expense of detailed granularity.

The first key assumption revolves around Homogeneous Elasticities. This assumption postulates uniform and unchanging elasticities (α,β,γ,δ) across various sectors and factors. By maintaining constant and identical elasticities across industries, labor skill levels, human capital, and natural resources, the model simplifies intricate variations in elasticities among different inputs.

Another fundamental simplification lies in the Presumption of Perfect Substitutability among inputs within the Constant Elasticity of Substitution (CES) production function. This assumption posits that inputs can be perfectly interchanged without altering the overall production function significantly, thereby simplifying the intricate interaction among inputs.

To further streamline the model, a decision is made to consolidate the diverse environmental impacts into a Single Aggregated Measure within the damage function (D). This simplification overlooks the complexities associated with distinct pollutants, opting instead for an aggregated measure of environmental impact.

In addition, the model employs Linear Investment Constraints instead of non-linear constraints to facilitate computational efficiency. While this simplification eases optimization complexities, it does so at the expense of a less detailed representation of real-world investment scenarios.

Furthermore, the assumption of Constant Total Factor Productivity (TFP) is introduced, implying a steady-state productivity without accounting for fluctuations due to technological advancements or other factors. This simplification aims to stabilize the analysis by assuming a constant level of productivity over time.

Moreover, the model operates within a framework of a Static Time Horizon, disregarding temporal fluctuations in environmental and economic parameters. This simplification enables a more straightforward analysis but foregoes the consideration of dynamic changes over time.

Lastly, the model assumes the absence of External Shocks or exogenous factors that might impact the model's parameters or system behavior. This simplification isolates the internal mechanisms of the model from unpredictable external influences.

In summation, these simplifying assumptions act as essential tools to streamline the multifaceted economic-environmental model, facilitating a more approachable framework for initial analysis and interpretation. However, it's crucial to acknowledge that these assumptions come with trade-offs, potentially oversimplifying real-world complexities and nuances in exchange for analytical manageability.

### Theoretical Analysis: Interpreting and Explaining Model Components

The theoretical underpinnings of the multifaceted economic-environmental model reveal intricate dynamics influenced by various components and assumptions. The model's structure integrates theoretical constructs with practical implications, reflecting the synthesis of economic and environmental theories.

Nested CES Production Function: At the core of the model lies the nested Constant Elasticity of Substitution (CES) production function, aligning with economic theory (Arrow *et al.,* 1961). The nested structure embodies a complex interplay among capital, labor, human capital, and natural resources, adhering to the principles of substitutability and complementarity among inputs (Boone & Mulder, 2019). The adoption of CES reflects a compromise between Cobb-Douglas and Leontief functions, facilitating flexible input substitution (Lagomarsino, 2021).

Damage Function and Environmental Impact: The incorporation of the damage function elucidates the environmental consequences of economic activities (Nordhaus, 1993). The model's approach to aggregating multiple pollutants into a unified measure aligns with environmental economics theories emphasizing the quantification of externalities (Dasgupta, 2001). The simplification of environmental impact echoes the practical challenges of quantifying diverse pollutant effects (Weitzman, 1974).

Optimization and Investment Constraints: The optimization process over a static time horizon, subject to linear investment constraints, resonates with optimization theory in economics (Bertsekas & Tsitsiklis, 1996). Linear constraints offer computational advantages despite oversimplifying real-world investment dynamics (Rockafellar & Wets, 1998).

Simplifying Assumptions: The introduction of homogeneous elasticities, perfect substitutability, and constant Total Factor Productivity aligns with the tradition of simplifying assumptions in economic modeling (Simon, 1955). These assumptions balance model complexity with analytical tractability (Lucas, 1976), although they may overlook crucial nuances in real-world dynamics.

External Shocks and Time Horizon: The exclusion of external shocks aligns with the concept of ceteris paribus in economic analysis (Keynes, 1936), allowing for the isolation of internal mechanisms within the model. The choice of a static time horizon reflects limitations in accounting for dynamic changes over time (Solow, 1956).

In essence, the model synthesizes diverse economic and environmental theories while employing simplifying assumptions to render complex dynamics more manageable for analysis. The blend of theoretical underpinnings and pragmatic simplifications serves as a foundation for understanding the intricate interplay between economic growth, environmental impact, and policy implications.

## Qualitative Insights from Mathematical Relationships

The mathematical relationships inherent in the economic-environmental model yield qualitative insights into the complex interplay between economic growth, environmental impact, and policy implications.

Interdependence of Inputs: The nested CES production function encapsulates the intricate relationships among inputs—capital, labor, human capital, and natural resources. The formulation showcases their interdependence, demonstrating how changes in one input influence overall output (Boone & Mulder, 2019). This highlights the necessity of balanced and efficient allocation of resources for sustained economic growth while underscoring the importance of environmental conservation.

Trade-offs and Environmental Impact: The CES function's elasticity parameters ($\alpha,\beta,\gamma,\delta$) elucidate trade-offs among different inputs. Higher elasticities imply greater responsiveness of output to changes in specific inputs. This insight into trade-offs aids in understanding how optimizing economic output might impact the environment—higher capital or labor usage might boost GDP but potentially exacerbate environmental degradation, emphasizing the need for sustainable development strategies (Dasgupta, 2001).

Environmental Implications of Investment: The constraints governing investment into various sectors—capital, labor, and human capital—impact environmental outcomes. The formulation illustrates how investment decisions affect resource utilization and emissions generation. Balancing investment across sectors to optimize social welfare necessitates considering environmental implications, emphasizing the need for eco-friendly investments and technologies (Perman *et al.,* 2011).

Optimization and Policy Trade-offs: The optimization process aims to maximize social welfare while adhering to investment constraints. The trade-offs revealed through this process underscore the challenges policymakers face. Balancing economic growth, environmental sustainability, and social welfare requires navigating intricate trade-offs and devising policies that mitigate environmental degradation without hampering economic progress (Dasgupta & Heal, 1979).

Simplifying Assumptions' Impact: The impact of simplifying assumptions on model outcomes is discernible. Homogeneous elasticities, perfect substitutability, and linear constraints streamline analysis but oversimplify real-world complexities. Acknowledging these simplifications offers insights into the limitations of the model and highlights areas where real-world complexities might differ significantly from model predictions (Simon, 1955).

In summary, the mathematical relationships embedded in the model offer qualitative insights into the nuanced relationships between economic activities, resource utilization, environmental impact, and policy trade-offs. These insights aid in understanding the multifaceted nature of sustainable development and serve as a basis for informed policy decisions and further empirical exploration.

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In essence, the model synthesizes diverse economic and environmental theories while employing simplifying assumptions to render complex dynamics more manageable for analysis. The blend of theoretical underpinnings and pragmatic simplifications serves as a foundation for understanding the intricate interplay between economic growth, environmental impact, and policy implications.

## CONCLUSIONS AND FUTURE DIRECTIONS

### Summary of Model Derivation

This paper presented a conceptual mathematical model integrating environmental economics principles and Sustainable Development Goal frameworks to holistically assess sustainability trade-offs. The model incorporates multidimensional CES production functions linking economic output to factor inputs of capital, labor, human capital, and natural resources. Environmental constraints via a damage function account for emissions and resource depletion. An optimization problem then seeks to allocate investments across sectors to maximize an intertemporal social welfare function consisting of GDP less environmental damages, subject to sustainability constraints on human, infrastructure, and natural capital accumulation.

While theoretical, this integrated structure establishes a foundation for capturing the complex interlinkages between economic activities, social welfare, and environmental sustainability. The model formalizes inherent trade-offs revealed through the optimization process and permits analyzing how balanced investment strategies can further sustainable development priorities.

### Potential Real-World Applications

With further empirical validation and calibration, the model could be applied to inform national policies or local planning for achieving SDGs. Data-driven analysis would determine optimal sectoral budget allocations that respect ecological limits while maximizing human well-being. The model also offers a tool to systematically assess trade-offs arising from competing economic interests, social needs, and environmental impacts within sustainable development processes.

### Recommendations for Future Refinement

The model would benefit from dynamic extensions capturing innovation, inequality, climate change, uncertainty, and validation against historical data. Incorporating these critical features would enhance realism and policy relevance. Overall, this integrated model provides a launching point to quantify the complex relationships governing sustainability - supporting the design of integrated pathways aligned with economic prosperity, social welfare, and environmental health.

I aimed to refine the language to more clearly summarize the key components derived, potential for application, and areas for valuable model enhancement. Please let me know if you would like any revisions or have additional suggestions for improving this concluding section.

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