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Managing Vertical Drainage Well Systems for Environmental Protection

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Abstract: Through systems of vertical drain wells with production rates or levels defined in them as function of time to create the artificially insulated surface, inside which one the field of contamination would be completely massed.

Keywords: Modeling, Non-Stationary Filtration, Stratified Porous Environment, Algorithm, Program, Ecology.

INTRODUCTION

In description of conduct of complex system exists several approaches, precisely: employment experience of researcher and intuition, comparison with given experiments which made in identical and resembling systems, at last mathematical modeling.

Naturally in studying of complex objects may employ from different approaches. Though in modern stage of scales and nature interference of person in natural ecological, systems so great, what intuition often leads. Experimentation of natural system is not usually possible. That's why construction of mathematical modeling of ecological system with rather degree of punctuality and realization such models actual.

THE MAIN RESULTS AND FINDINGS

Thus, for example, in investigation of ecological system in soils, petroleum's, deposits of gas and in atmosphere. It is taken models which we can take an effective approximate-analytical decision. It is known that in result of intensive development of agriculture, specifically cotton-growing, industry, also rapid growth of towns in ecological situation in Uzbekistan demands global appreciation at monitoring of surroundings. It is rather indicate that the surface and subterranean water springs are continuing to get dirty with agriculture mineral manure, poison chemicals, strong mineralized and dirty collector-drainage water. Imperfection of traditional methods of watering and drawing into agriculture liable to salt of earth brings to

that the recourses of saline reimbursement water in republic reached considerable areas.

Nowadays nature of contamination of underground waters reaches threatening sizes, that are well enough circumscribed in [1-7], where is showed, that the processes of contamination can be justified by the sufficiently general physical and mathematical theories and as indispensable practical basis of the tendered methods. As against the approaches reviewed earlier, with the view a preservation of the environment from propagation of a various sort of contamination of underground waters the mathematical model and her analytical solution is offered.

Through systems of vertical drain wells with production rates or levels defined in them as function of time to create the artificially insulated surface, inside which one the field of contamination would be completely massed.

It is possible to contour the real seams drained by wells at making the insulated fields by their surfaces of the various geometrical shape under certain conditions on its boundary, in particular, by barrel of radius R .

For this purpose the field of filtering $0 < r \leq R$ is considered, inside which one there is a field of contamination located in a zone $0 < r < R_{T1}$. The centre of contamination is formed, for example, by well by force of injection of various toxic matters with a production rate $Q_c(t)$ (in this case $Q_c(t) < 0$) in well water-permeable horizon. The partially furnished

harmful matters filtered through a porous medium, are intercepted by the vertical well row with an aggregate production rate Q_{r1} . It is a system of wells is disposed on a circumference $r = R_1$. For prevention of propagation of toxic matters in a zone $r = R_1 > R_{r1}$ with a rather favourable ecological situation it is offered to create the insulated surface by well rows uniformly located on a circumference $r = R_{r2} > R_1$, with an

$$\left. \begin{aligned} \frac{1}{a_1} \frac{\partial S_1}{\partial t} &= \frac{\partial^2 S_1}{\partial z^2}, & S_1(x, z, t), & \quad m_2 + m_3 < z < m = \sum_{i=1}^3 m_i, \\ \frac{1}{a} \frac{\partial S}{\partial t} &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial S}{\partial r} \right) + \frac{K_1}{T} \frac{\partial S_1}{\partial z} \Big|_{z=m_2+m_3} - \frac{K_2}{T} \frac{\partial S_2}{\partial z} \Big|_{z=m_3}, \\ \frac{1}{a_2} \frac{\partial S_2}{\partial t} &= \frac{\partial^2 S_2}{\partial z^2}, & 0 < z < m_3, \end{aligned} \right\}$$

The conventional labels here are used.

Let's subordinate this system to following boundary requirements.

$$S_1(r, z, 0) = S(r, 0) = S_2(r, z, 0) = 0 \quad H_1(\tau)$$

$$\left(\alpha_1 S_1 - \beta_1 \frac{\partial S_1}{\partial z} \right)_{z=m} = g_1(t),$$

$$S_1(r, m_2 + m_3, t) = S(r, t),$$

$$\lim_{r \rightarrow 0} r \frac{\partial S}{\partial r} = -\frac{Q_c(t)}{2\pi T},$$

$$\frac{\partial S}{\partial r} \Big|_{r=R_{rj}+0} - \frac{\partial S}{\partial r} \Big|_{r=R_{rj}-0} = -\frac{Q_{rj}(t)}{2\pi T R_{rj}} \quad (j=1,2; \quad t > 0),$$

$$\left(\alpha_2 S + \beta_2 \frac{\partial S}{\partial r} \right)_{r=R_2} = g_2(t),$$

$$S(r, t) = S_2(r, m_3, t),$$

$$\left(\alpha_3 S_2 + \beta_3 \frac{\partial S_2}{\partial z} \right)_{z=0} = g_3(t).$$

The solution of the given boundary value problem exists and is unique. And at given production rates $Q_c(t), Q_{rj}(t), j=1,2$ in field $R_{r1} < r < R_{r2}$ the insulated surface is formed, which one during time is displaced in room. The law of this transition is governed through a production rate $Q_{r2}(t)$. In particular, by its controlling, it is possible to create the fixed insulated surface, on which one at $r = R_1 + 0$ and $r = R_1 - 0$ will be fulfilled following requirements

$$S(R_1 + 0, t) = S(R_1 - 0, t),$$

$$\frac{\partial S}{\partial r} \Big|_{r=R_1+0} = \frac{\partial S}{\partial r} \Big|_{r=R_1-0} = 0.$$

Let's illustrate it on a following simple example.

aggregate controllable production rate $Q_{r2}(t)$ or level $H(t)$, which one is necessary for determining in further. Usually for definition of lowering of heads in stratified porous mediums the equations set in partial derivatives of a parabolic type is used. So, for example, in the following system for a triplex seam was reviewed.

With this purpose the following mathematical model is considered, which one is a special case of model reduced in [6-10]:

$$\frac{\partial S}{\partial \tau} = \frac{1}{\sigma} \frac{\partial}{\partial \sigma} \left(\sigma \frac{\partial S}{\partial \sigma} \right) - AS, \quad (1)$$

$$S(\sigma, 0) = 0, \quad (2)$$

$$\lim_{\sigma \rightarrow 0} \sigma \frac{\partial S}{\partial \sigma} = -\frac{Q_c(\tau)}{2\pi T}, \quad (3)$$

$$\frac{\partial S}{\partial \sigma} \Big|_{\sigma=\sigma_{r1}+0} - \frac{\partial S}{\partial \sigma} \Big|_{\sigma=\sigma_{r1}-0} = -\frac{Q_{r1}(\tau)}{2\pi T \sigma_{r1}}, \quad (4)$$

$$\frac{\partial S}{\partial \sigma} \Big|_{\sigma=\sigma_1} = 0 \quad (5)$$

Where

$$t^* = \frac{R_2^2}{a}, \quad A = A*t^*, \quad \sigma_{r1} = \frac{R_{r1}}{R_2}, \quad \sigma_1 = \frac{R_1}{R_2}$$

Requirement (4) is replaceable on an equivalent requirement $S(\sigma_{r1}, \tau) = H_1(\tau)$ where $H_1(\tau)$ in this case unknown function, which is immediately bound up with production rate of the row $Q_{r1}(\tau)$.

After substitution of a requirement (4) on $S(\sigma_{r1}, \tau) = H_1(\tau)$ we can use to (1) - (5) integrated Laplace's transformation, then in view of a requirement (2) we shall receive in the field of the image the Bessel's equation

$$\frac{1}{\sigma} \frac{d}{d\sigma} \left(\sigma \frac{d\bar{S}}{d\sigma} \right) - \omega^2(p) \bar{S} = 0 \quad (6)$$

Where $\omega^2(p) = p + A$ p- Argument of Laplace's transformation.

The solution of the equation (6) looks like

$$\bar{S} = c_1 I_0(\omega \sigma) + c_2 K_0(\omega \sigma)$$

After definition c_1 and c_2 from requirements

$$\lim_{\sigma \rightarrow 0} \sigma \frac{d\bar{S}}{d\sigma} = -\frac{\bar{Q}_c}{2\pi T}$$

$$\bar{S}(\sigma_{r1}, p) = \bar{H}_1(p) \div H_1(\tau)$$

We will receive $\sigma_{r1} \leq \sigma \leq \sigma_1$

$$\bar{S} = \frac{\bar{H}_1 I_0(\omega\sigma)}{I_0(\omega\sigma_{r1})} - \frac{\bar{Q}_c}{2\pi T I_0(\omega\sigma_{r1})} [I_0(\omega\sigma)K_0(\omega\sigma_{r1}) - K_0(\omega\sigma)I_0(\omega\sigma_{r1})] \tag{7}$$

Now shall discover solution in field $\sigma_{r1} \leq \sigma \leq \sigma_1$, which one is searched as:

$$\bar{S} = c_3 I_0(\omega\sigma) + c_4 K_0(\omega\sigma)$$

The constants c_3 and c_4 are defined from requirements

$$\bar{S}(\sigma_{r1}, p) = \bar{H}_1(p), \quad \left. \frac{d\bar{S}}{d\sigma} \right|_{\sigma=\sigma_1} = 0$$

In outcome we shall discover

$$\bar{S} = \frac{\bar{H}_1}{\Delta^*} [I_0(\omega\sigma)K_1(\omega\sigma_1) + I_1(\omega\sigma_1)K_0(\omega\sigma)] \tag{8}$$

Where $\Delta^* = I_0(\omega\sigma_{r1})K_1(\omega\sigma_1) + I_1(\omega\sigma_1)K_0(\omega\sigma_{r1})$

The dependence between lowering of wells row level at $\sigma = \sigma_{r1}$ and production rates of wells \bar{Q}_c and \bar{Q}_{r1} in the field of the image will turn out if to use a requirement

$$\left. \frac{\partial \bar{S}}{\partial \sigma} \right|_{\sigma=\sigma_{r1}+0} - \left. \frac{\partial \bar{S}}{\partial \sigma} \right|_{\sigma=\sigma_{r1}-0} = -\frac{\bar{Q}_{r1}(p)}{2\pi T \sigma_{r1}}$$

As a result of that we will have

$$\bar{H}_1 = \frac{\Delta^*}{2\pi T I_1(\omega\sigma_1)} \left[\bar{Q}_c + \bar{Q}_{r1} I_0(\omega\sigma_{r1}) \right] \tag{9}$$

Substituting (9) in (7) and (8), we shall receive accordingly following solutions in field $0 < \sigma \leq \sigma_{r1}$ and $\sigma_{r1} \leq \sigma \leq \sigma_1$

$$\bar{S} = \frac{\Delta^*}{2\pi T I_1(\omega\sigma_1)} \left[\bar{Q}_c + \bar{Q}_{r1} I_0(\omega\sigma_{r1}) \right] \frac{I_0(\omega\sigma)}{I_0(\omega\sigma_1)} - \frac{\bar{Q}_c}{2\pi T I_0(\omega\sigma_1)} \times [I_0(\omega\sigma)K_0(\omega\sigma_1) - I_0(\omega\sigma)K_0(\omega\sigma_{r1})] \\ \bar{S} = \frac{\bar{Q}_c + \bar{Q}_{r1} I_0(\omega\sigma_{r1})}{2\pi T I_1(\omega\sigma_1)} [I_0(\omega\sigma)K_1(\omega\sigma_1) + I_1(\omega\sigma_1)K_0(\omega\sigma)] \tag{10}$$

From (10) the head lowering on the insulated surface is determined at $\sigma = \sigma_1$: $\bar{S} = \bar{F}_1(p)$, where

$$\bar{F}_1(p) = \frac{\bar{Q}_c + \bar{Q}_{r1} I_0(\omega\sigma_{r1})}{2\pi T \omega \sigma_1 I_1(\omega\sigma_1)} \tag{11}$$

If to set values of the rates of flux $Q_c(t)$ and $Q_{r1}(t)$ the head lowering on the insulated surface is determined from (11).

We shall esteem a case, when Q_c and Q_{r1} are constants, then the lowering can be calculated levels at $r = R_1$ by use the following equation

$$S_1(\sigma_1, \tau) = \frac{Q_c + Q_{r1} I_0(\sqrt{A}\sigma_{r1})}{2\pi T \sigma_1 \sqrt{A} I_1(\sqrt{A}\sigma_{r1})} - \frac{Q_c - Q_{r1}}{\pi T \sigma_1 A} e^{-A\tau} - \frac{e^{-A\tau}}{\pi T} \sum_{n=1}^{\infty} \frac{Q_c + Q_{r1} J_0\left(\frac{\xi_n \sigma_{r1}}{\sigma_1}\right) - \frac{\xi_n^2}{\sigma_1^2} \tau}{(\xi_n^2 + A\sigma_1^2) J_0(\xi_n)} e^{-\frac{\xi_n^2}{\sigma_1^2} \tau},$$

Where ξ_n positive root of the equation $J_1(\xi) = 0$. $\tag{12}$

In particular, from a requirement $S(\tau_1, 0) = 0$ the following sums of series are determined

$$\sum_{n=1}^{\infty} \frac{1}{(\xi_n^2 + \sigma_1^2 A) J_0(\xi_n)} = \frac{1}{\sigma_1 A} \left[\frac{\sqrt{A}}{2I_1(\sqrt{A}\sigma_1)} - 1 \right], \\ \sum_{n=1}^{\infty} \frac{J_0\left(\frac{\xi_n \sigma_{r1}}{\sigma_1}\right)}{(\xi_n^2 + \sigma_1^2 A) J_0(\xi_n)} = \frac{1}{\sigma_1 A} \left[\frac{\sqrt{A} I_1(\sqrt{A}\sigma_{r1})}{2I_1(\sqrt{A}\sigma_1)} - 1 \right]$$

Let's discover now in field $\sigma_1 \leq \sigma \leq \sigma_{r2} = \frac{R_{r1}}{R_2}$ a solution of a boundary value problem

$$\frac{\partial S}{\partial \tau} = \frac{1}{\sigma} \frac{\partial}{\partial \sigma} \left(\sigma \frac{\partial S}{\partial \sigma} \right) - AS, \quad S(\sigma, 0) = 0, \\ S(\sigma_1, \tau) = S_1(\sigma_1, \tau), \quad S(\sigma_{r2}, \tau) = S^*(\sigma_{r1}, \tau),$$

In which one the function $S^*(\sigma_{r1}, \tau)$ introduces lowering of well row level at $r = R_{r2}$ and is subject to definition.

After association of a side condition $\left. \frac{\partial S}{\partial \sigma} \right|_{\sigma=\sigma_1} = 0$,

The following relation is gained:

$$S^*(\sigma_{r2}, t) = \frac{Q_c + Q_{r2} I_0(\sqrt{A}\sigma_{r1})}{2\pi T} [I_0(\sqrt{A}\sigma_{r2})K_1(\sqrt{A}\sigma_1) + K_0(\sqrt{A}\sigma_{r2})I_1(\sqrt{A}\sigma_1)] - \frac{Q_c + Q_{r2}}{\pi T A \sigma_1^2} e^{-A\tau} + \frac{e^{-A\tau}}{2\sigma_1 T} \sum_{n=1}^{\infty} \frac{\xi_n \left[Q_c + Q_{r1} J_0\left(\frac{\xi_n \sigma_{r1}}{\sigma_1}\right) \right]}{(\xi_n^2 + A\sigma_1^2) J_0(\xi_n)} \times \left[J_0\left(\frac{\xi_n \sigma_{r2}}{\sigma_1}\right) Y_1(\xi_n) - J_1(\xi_n) Y_0\left(\frac{\xi_n \sigma_{r2}}{\sigma_1}\right) \right] e^{-\frac{\xi_n^2}{\sigma_1^2} \tau}$$

Using a requirement $S^*(\sigma_{r2}, 0) = 0$, we shall receive the similarly to previous value of the following sums of series

$$\sum_{n=1}^{\infty} \frac{\xi_n \Omega(\xi_n)}{(\xi_n^2 + \sigma_1^2 A) J_0(\xi_n)} = \frac{2}{\pi A \sigma_1} - \frac{\sigma_1}{\pi} \Phi(A),$$

$$\sum_{n=1}^{\infty} \frac{\xi_n \left(J_0 \frac{\sigma_{r1}}{\sigma_1} \right) \Omega(\xi_n)}{(\xi_n^2 + \sigma_1^2 A) J_0(\xi_n)} = \frac{2}{\pi A \sigma_1} - \frac{\sigma_1 I_0(\sqrt{A} \sigma_{r1})}{\pi} \Phi(A),$$

Where

$$\Omega(\xi_n) = J_0 \left(\xi_n \frac{\sigma_{r2}}{\sigma_1} \right) Y_1(\xi_n) - J_1(\xi_n) Y_0 \left(\xi_n \frac{\sigma_{r2}}{\sigma_1} \right),$$

$$\Phi(A) = I_0(\sqrt{A} \sigma_{r2}) K_1(\sqrt{A} \sigma_1) + K_0(\sqrt{A} \sigma_{r2}) I_1(\sqrt{A} \sigma_1)$$

ξ_n – Positive roots the equation (12).

In outcome by maintaining lowering of well row level as functions of time t, located on a circumference of radius $r = R_{r2}$, there is an impervious cylindrical surface of radius $r = R_1$.

CONCLUSION

Generally the combination of approximately - analytical and numerical computational methods is used [6-10] for constructing the insulated fields of contamination in composite fields of filtering composed of several interacting stratum with various water-permeable properties.

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